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Second Exam MTH 221 , Summer 011

Ayman Badawi

QUESTION 1. (Each = 1.5 points, Total = 12 points) Answer the following as true or false: NO WORKING NEED BE SHOWN.

(i) $dim(P_7) = 7$.

(ii) Every 5 points in R^4 are dependent

(iii) Every 4 points in R^5 are independent

(iv) Every 6 polynomials in P_6 form a basis for P_6

(v) The point $(2, 2, 2, 10) \in span\{(1, 1, 1, 1)\ ,\ (-1, -1, -1, 7)\}$

(vi) If v_1, v_2 are independent points in R^3 and $v_3 \notin span\{v_1, v_2\}$, then $\{v_1, v_2, v_3\}$ is a basis for R^3

(vii) If v_1, v_2, v_3 are dependent points in R^5, then $v_1, -3v_1 + v_2, v_3$ are also dependent points in R^5.

(viii) If $D = span\{(2, -2, 0)\ ,\ (-2, 2, 1),\ (4, -4, 1)\}$, then $dim(D) = 2$

QUESTION 2. (Each = 2 points, Total = 28 points) Circle the correct letter for each of the questions below:

(i) One of the following statements is correct

a. $(2, 0, -2), (-2, 10, 20), (-4, -10, 1)$ are independent
b. (2, 1), (0, 1), 1,0) are independent
c. (0, 1, 1), (-1, 2, 0), (-1, 3, 1) are independent
d. $\{(2, 1), (1, 0.5)\}$ is a basis for R^2.

(ii) Let $F = \{(3a + b, 0, 6a + 2b) \mid a, b \in R\}$. We know that F is a subspace of R^3. Then $dim(F) =$

a. 2 b. 3 c. 1 d. cannot be determined

(iii) Consider F in the previous question. Then one of the following points does not belong to F.

a. (1, 0, 2) b. (2, 0, 4) c. (3, 0, 5) d. (0, 0, 0)

(iv) Given v_1, v_2, v_3 are independent points in R^{10}. One of the following statements is correct:

a. $v_1, v_2, 2v_1 + v_3$ are independent points in R^3
b. $v_1, v_1 + v_3, v_3$ are independent points in R^3
c. $v_1, v_1 + v_2, -3v_1 + v_2$ are independent points in R^3
d. All previous statements are correct.

(v) Let $D = span\{(1, 1, 1, 1), (-1, -1, -1, 0), (0, 0, 0, 2), (-1, -1, -1, 4)\}$. One of the following is a basis for D

a. $B = \{(1, 1, 1, 1), (0, 0, 0, 1)\}$
b. $B = \{(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)\}$
c. $B = \{(1, 1, 1, 1), (-1, -1, -1, 0), (0, 0, 0, 2)\}$
d. None of the previous is correct

(vi) Let $H = \{a + (b + 2c)x + (3b + 6c)x^2 \mid a, b, c \in R\}$ be a subspace of P_3. Then $dim(H) =$

a. 3 b. 1 c. 4 d. 2

(vii) Let H as in the previous question. Then one of the following is a basis for H

a. $B = \{1, x, 3x^2\}$
b. $B = \{1, x + 3x^2, 3x + 6x^2\}$
c. $B = \{1, x, 2x, 3x^2, 6x^2\}$
d. $B = \{1, x + 3x^2\}$.

(viii) Let $A = \begin{bmatrix} a_1 & 2 & 4 \\ a_2 & 4 & 8 \\ a_3 & -4 & -7 \end{bmatrix}$ such that det(A) = 20. The value of x_1 in solving the system $AX = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ is

a. 5 b. 10 c. 0.2 d)Can not be determined

(ix) Consider the previous question, the value of x_2 in solving the system $AX = \begin{bmatrix} 4 \\ 8 \\ -7 \end{bmatrix}$ is

a. 0 b. 1 c. 0.1 d) None of the previous is correct

(x) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 2 & 1 \\ -1 & -1 & -2 & 2 \\ -1 & -1 & -1 & -2 \end{bmatrix}$ the (3, 4)-entry of A^{-1} is

a. -3 b. -2 c. 0.2 d. 2 e. None of the previous is correct

(xi) Given A is a 2×2 matrix such that $\begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix} + 2A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$. Then $A =$

a. $\begin{bmatrix} 1 & -1 \\ -5 & -4 \end{bmatrix}$ b. $\begin{bmatrix} 9 & 5 \\ -2 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 2 & -1 \\ -9 & 5 \end{bmatrix}$ d. None of the previous is correct

(xii) Given $\left(A^T \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \right)^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$. Then $A =$

a. $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ b. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ c. $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ d. None of the previous is correct

(xiii) One of the following is a subspace of R^3:

a. $\{(a, 2a - b^2, 0) \mid a, b \in R\}$ b. $\{(a + b, -2a, b) \mid a, b \in R\}$

c. $\{(3, -a, -b) \mid a, b \in R\}$ d. $\{0, ba + a, -2b) \mid a, b \in R\}$

(xiv) One of the following is a subspace of P_3:

a. $\{3 - ax + ax^2 \mid a \in R\}$ b. $\{3ax + ax^2 \mid a \in R\}$

c. $\{x + ax^2 \mid a \in R\}$ d. $\{a + ax + x^2 \mid a \in R\}$

QUESTION 3. (4 points) Find a basis for R^4, say B, such that B contains the two independent points $(3, 3, -4, 0), (-6, -6, 9, 3)$.

QUESTION 4. (6 points) Let $A = \begin{bmatrix} 4 & -6 & 5 \\ -8 & 13 & -8 \\ -4 & 5 & -6 \end{bmatrix}$

a) Find the LU-Factorization of A.

b) Use (a) to solve the system $AX = \begin{bmatrix} 8 \\ -11 \\ -11 \end{bmatrix}$

Faculty information

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